First International Congress on Tools for Teaching Logic. Learning Logic with Smullyan The advantages of Smullyan's Tree calculus for teaching logic, and a program for teaching and testing derivations: SM-Tutor

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Abstract

Smullyan's tree calculus for first order logic has a number of advantages for teaching logic. It paves the way to short proofs because it enjoys the subformula property. For propositional calculus it gives a decision procedure and allows to draw information from unsuccessful derivations. For predicate calculus it can be made to deliver finite counter models, if they exist. And it is easily adaptable to modal calculi such as T, S4 and S5 - also in their predicate logic versions.
We present SM-TUTOR, a program for testing derivations and teaching how to arrive at derivations in the above mentioned calculi. It offers context sensitive help. The program is window-oriented and written in Tcl/Tk in order to be platform-independent.

1 The advantages of Smullyan's Tree calculus for teaching logic

1.1 Propositional calculus

Smullyan's tree calculus has been developed out of Gentzen's sequent calculus. It shares with its ancestor the subformula property which guarantees derivations which only contain subformulae of the target formula and thus avoids"deviations" such as cuts. Although Smullyan's system was mainly developed to yield a perspicuous completeness proof it has been found useful for teaching logic. The main reason is, that the student is provided with clear directions for …nding derivations. Still there is room for "good" and "bad" strategies for …nding them.

1.2 Propositional calculus

In the propositional case we get a simple decision procedure. More than that ! Once a derivation-tree has been completely developed, one is able to use the tree to "read o¤" the falsifying and a fortiori the verifying combinations of values for the variables. A simple variant of the calculus can be used to …nd directly the values compatible with the formula's truth. The rules of the calculus can be motivated directly from the truthtables for the propositionala connectives.

1.3 Predicate calculus

The predicate calculus, of course, does not admit a decision procedure. A parsimonious and intelligent use of the instantiation rules, however, permits to hit upon a …nite model of the negation of the target formula, in case such …nite models exist.

1.4 Modal calculi

We treat the modal calculi T, S4, S5. We proceed by numerating the worlds in a Tractatus-like way, in the sense that world n1.n2 "sees" world n1.n2.n3.

The relational properties of the visibility (accessibility-) relations chosen determine the resulting calculus in the usual way: no restrictions =>T, Transitivity =>T4, universal visibility =>S5. We extend modal logic to predicate logic in the straightforward way. More sophisticated possibilities have not (yet) been considered.

2 The program "SM-Tutor"

If given the possibility we want to show our program. We want to do a demonstration.

Contrary to the preconceptions of some logic novices our "SM" is not derived from "Sado-Masochism" but from "SMULLYAN".

The program exists in two versions, a DOS- and a Windows version. It can cope with propositional, predicate and modal logic. There are three modi - modes of employment - of the program: 1. checking a (step in a) derivation, 2. proposing the next step to be done, 3. attempting to …nd and show a complete derivation of the given target formula.

The sreen is used as a window which can be moved across the derivation tree. The user is freed from the task of formatting the tree - he does however have the obligation to indicate where the next formula of the derivation in work has to be appended.

The DOS-version was written in Pascal, the Windows version in Tcl/Tk. Our aim in chosing these latter languages was platform independence. This aim, however, not yet been fully achieved.

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2.1 Reference

Raymund Smullyan, First Order Logic,