First International Congress on Tools for Teaching Logic Diagrams and Proofs

Horacio Faas
horacio@goedel.¤yh.unc.edu.ar
horacio@…loso…a.uncor.edu
Universidad Nacional de Córdoba
Argentina

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It is well known that the use of diagrams has been traditionally considered as an auxiliary tool that doesn't deserves any main place in mathematical proofs. However, in many cases diagrammatical thinking has guided discoveries and demonstrations. Accepting that there are several representation systems, among which the more frequent two are the linguistic one and the diagrammatic one, one can wonder if anyone of them is better than the other to yield valid inferences. Con…ning us to these two systems, we shall call a system pure if it always makes inferences within the system, vgr. from sentences to sentences if linguistic and from diagrams to diagrams if diagrammatic. If while inferring it sometimes passes from one system to the other, it is called heterogeneous. There are thence four possibilities for premise and conclusion of the inference: 1) from sentence to sentence, 2) from sentence to diagram, 3) from diagram to sentence, 4) from diagram to diagram. Case 1 is usual in logic, pure sentential systems, the others aren't. A computer program has been recently developed that copes with cases 2 and 3, and case 4 is a rather odd one. As far as I know, there have been very few approaches to this last case, some of which I'll criticize in this paper. For instance, the one presented by Furnas in Reasoning with diagrams only has some inconveniencies only surmounted by appealing to another representation. I'll focus on the paramount role played by diagrams in proving some well known theorems, such as Euler's formula for polyhedra and I'll intend to show how Cauchy's proof for that formula may be recapituleted by using some of the rules of the computer program Hyperproof, specially those related with proof by cases: Merge, Exhaust and Inspect. The proof begins taking out one face of the polyhedron -assume it is a cube- to let it become (1) a ‡at …gure topologically equivalent with the "open" cube, then dividing each face in two triangles each one of them, in the whole …gure, to be gradually (2) picked out till one arrives to the last lonely triangle. In Hyperproof one may take (1) to be an initial situation and each one of steps (2) as a new derived situation. But here is where proof by cases applies: each time you pick out a face you are taking out a side and nothing else, or you are taking out two sides and one vertex. For Hyperproof this is Cases Exhaustive. A proof in Hyperproof yields a final situation that implies every preceding situation in that proof. Similarly, the relations amidst the number of sides, faces and vertex in the final triangle in Cauchy's proof implies those types of relations in preceding figures. Both processes use heterogeneous inferences.