ADN (Asistente para Deducción Natural - "Natural Deduction Assistant")

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1.- INTRODUCTION

Logic provides calculus methods that allow us to infer, by syntactic handling, new formulas from others formulas already known. One of these methods is Natural Deduction, whose reasoning process is very close to that of humans. This method allows us to obtain new formulas from others already given by means of simple rules. In this paper, we present a didactic tool for aiding in the learning process of Natural Deduction. This tool has been developed in the Department of Computer Science and Artificial Intelligence and it is applied during the practical sessions of the obligatory subject *First Order Logic*, belonging to the first year of Computer Engineering in the University of Alicante [Llorens96] and [Llorens98]. This development is a tool for helping students to write logic well-formed formulas and to make proofs. Other tools can also be useful when learning Logic [Goldson93], [Barwise92], [Barwise94] and [Allwein96].

2.- THE LANGUAGE

Logic tries to formalise human knowledge. Such knowledge is acquired and transmitted by means of a language. But human natural language is ambiguous and complicated, and so, an artificial language is required to work formally with knowledge. In the language used in First Order Logic, sentences are decomposed in objects and relationships/properties, obtaining the concepts of terms and predicates as a result. For instance, let “there is a woman that is loved by every man” be a sentence. A well-formed formula (wff) in the language of first order logic could be:

$$\exists y \, \{ \, woman(y) \wedge \forall x \, [man(x) \rightarrow loves(x,y)] \, \}$$

Predicates can be expressed by letters: P (woman), Q (man) and R (loves), obtaining the next expression instead:

$$\exists y \, \{ \, P(y) \wedge \forall x \, [Q(x) \rightarrow R(x,y)] \, \}$$

A wff can be represented as a labelled tree in order to distinguish the syntactic structure of the formula. Our system ADN provides labelled trees, such as the one shown in Figure 1, representing the previous wff.

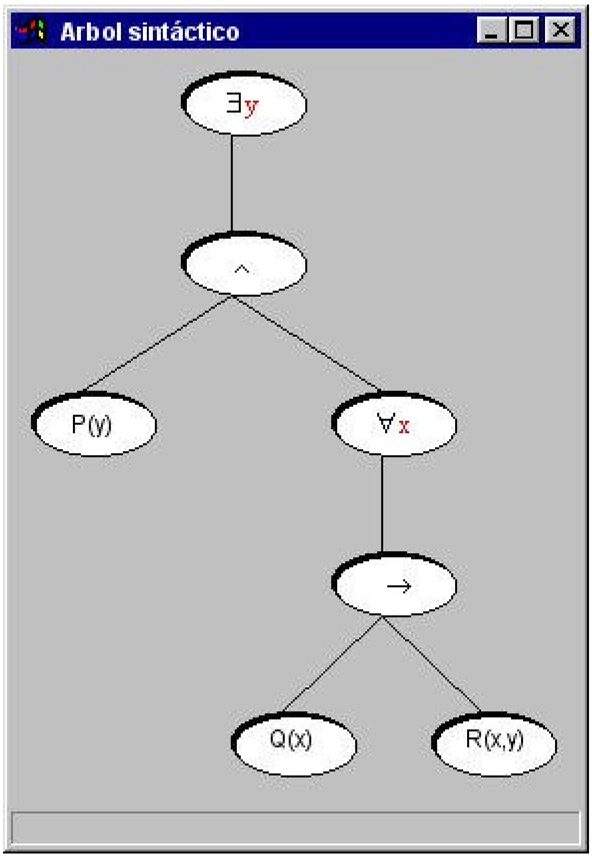

Figure 1 : Syntactic labelled tree

3.- NATURAL DEDUCTION

One of the main aspects in Logic is the fact that, apart from a language to represent knowledge, it provides some reasoning or inference techniques that allow us to obtain new knowledge. In particular, we will use *Natural Deduction* [Garrido95],[Reeves90], which is a formal system that can arrive to certain conclusions from some premises and just some basic rules. From this point of view, if premises are assumed and every elemental step is justified by a basic rule, we will obtain new logical formulas that can be considered as conclusions derived from the premises. A deduction can be seen as an algorithm that obtains some outputs (conclusions) from some inputs (premises), using a set of given instructions (rules) [Llorens99].

Basic Rules

In order to determine the *basic rules* of the Natural Deduction, we will base in Sequent Calculus introduced by Gentzen [Gentzen34] that proposes two rules (one for introduction and another for elimination) for every logical symbol (connectives and quantifiers). If a connective or a quantifier that does not appear in premises is introduced in the conclusion of a basic rule, it will be an introduction rule; nevertheless if a connective or a quantifier that does appear in premises is eliminated from the conclusion of a basic rule, it will be an elimination rule. Intuitively, it can be seen that, if some procedures to add or eliminate the various logical symbols are available, premises can be transformed to the conclusion by simple syntactic operation. From an engineering point of view, we are dealing with taking apart the logical formulas that make up the premises in order to obtain its basic components (atomic formulas). Then, the atomic formulas are built in the appropriated configuration to obtain the desired conclusion. Rules can be considered as tools that allow us to build and to take apart such logical formulas.

	Introduction rule	Elimination rule
$\wedge$ (Conjunction)	IC	EC
$\vee$ (Disjunction)	ID	ED*
$\neg$ (Negation)	IN*	EN
$\rightarrow$ (Implication)	II*	EI*
$\forall$ (Universal quantifier)	IU	EU
$\exists$ (Existential quantifier)	IE	EE

* These rules are known by specific names:

ED	Proof by cases	IN	Reductio ad absurdum
II	Theorem of deduction	EI	Modus Ponens

Each step in a deduction scheme (and so, each logical formula written in this step) will be "justified" by the application of a basic rule to one or several previous steps.

Subdeductions

Another important aspect in Natural Deduction is the concept of subproofs (subdeductions or subderivations). In any step of the deduction scheme, a provisional assumption can be introduced, although it must be cancelled later. From the assumption to its cancellation we have a subdeduction. Provisional assumptions are a very powerful tool because it allows us to suppose whatever we want. However, there is a price to pay for it: to finish any demonstration every assumption must have been cancelled. So, the cancellation of provisional assumptions is a key aspect in Natural Deduction.
Using subdeductions allow us to "modularize" our deductions, decomposing the final objective in simpler ones that will take us to the desired conclusion.

4.- EXAMPLE

Let us consider an example of Natural Deduction. We assume that "there is a woman that is loved by every man", which is our premise. As we have already shown, it can be written in the language of the first order logic in the following way:

$$\exists y \{ P(y) \wedge \forall x [Q(x) \rightarrow R(x,y)] \}$$

We are dealing with demonstrating that from this premise it can be deduced that "every man loves some woman", which can be written as a well-formed formula as follows:

$$\forall x \{ Q(x) \rightarrow \exists y [P(y) \wedge R(x,y)] \}$$

In Figure 2, the ADN natural deduction is shown.

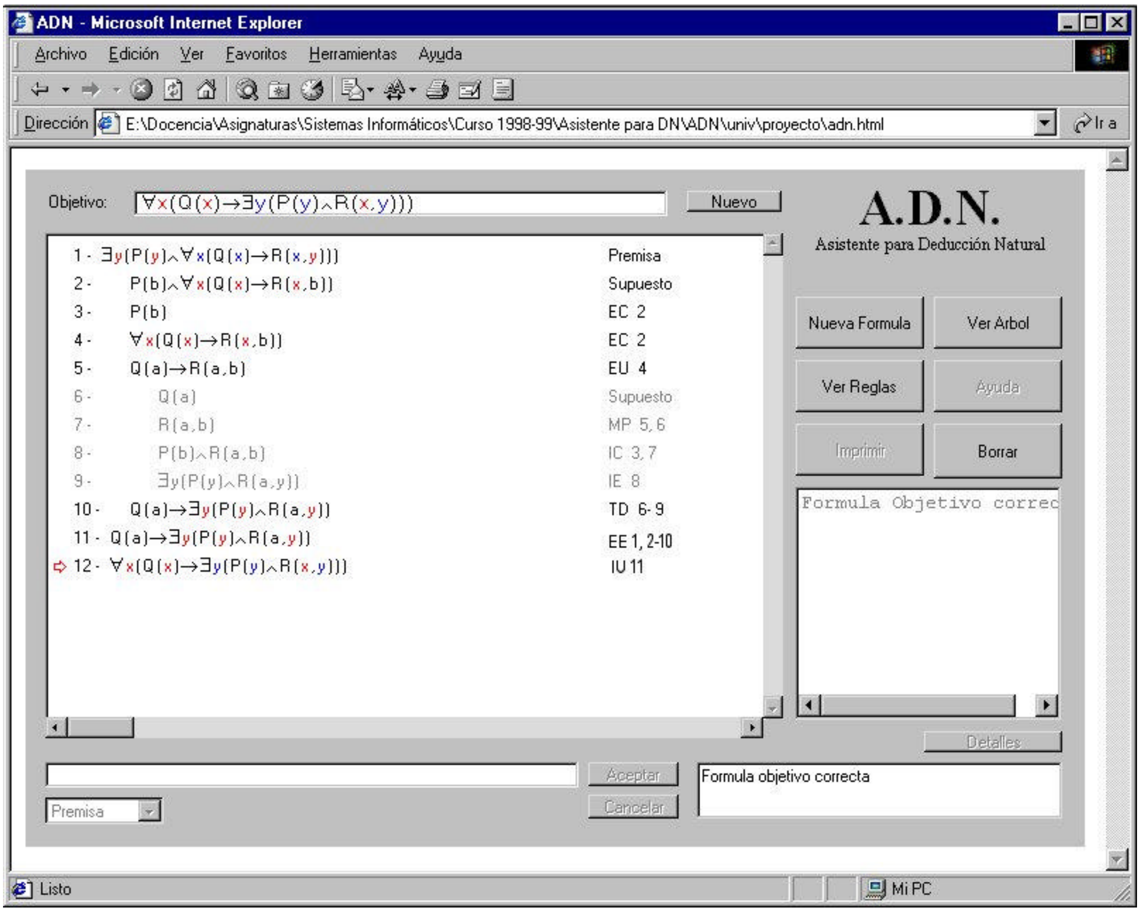

Figure 2: Natural Deduction

* There are two constraints for the elimination of the existential quantifier:

- The generic individual chosen in line 2, *b*, does not appear in any previous premise or assumption which must be cancelled.
- In the formula in which cancellation is applied (line 10) the generic individual *b* does not appear.

** There is also a constraint to the introduction of the universal quantifier: the individual that must be generalised, *a*, does not appear in any previous premise or assumption which must be cancelled.

In the previous deduction scheme, three areas can be observed (disposed in columns):

1. Numbered lines, to make references easier.
2. Logical formulas that are obtained. Indentation to the right is used to show that a new assumption is stated; indentation to the left shows that the assumption is cancelled. So, assumption in line 2 is cancelled in line 11, so that lines between 2 and 10 make up a subproof. Following the same scheme, assumption in line 6 is cancelled in line 10, that is, lines between 6 and 9 make up another subproof.
3. The formula that is obtained is justified by the application of a basic rule to one or several previous formulas. For instance, *EE 1,2-10* means that this formula is obtained because there is a existentially quantified formula in line 1, in line 2 a generic individual is supposed to satisfy such formula, and in line 10 we arrive to a conclusion that does not depend on the election. So, formula in line 11 can be deduced.

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