First International Congress on Tools for Teaching Logic
Teaching Logical reasoning in high school

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Abstract

This report deals with two general features in the usual curriculum in high school. On the one hand there is no explicit inclusion of "Logical Reasoning" in secondary studies, on the other hand it is commonly admitted that the logical reasoning is a crucial part of general education in any of the areas most closely associated with Logic: Philosophy, Mathematics, Computer Science, etc.
Logic courses are present in the college level, in Philosophy, Mathematics, Computer Science and Linguistics courses; and there are not doubt that logical reasoning is in the basis of any subject related with Logic. But the situation is di¤erent in the pre-college curriculum, whereas other …elds are explicit in the secondary curriculum, Logic seems to have been left outside it.
In this paper I …rst explain the presence of logical reasoning taught in the high school curriculum (mainly as a Logical mathematical reasoning). Secondly I claim for the recognition of Logic as a subject matter of the secondary curriculum in its own right.

1 Logical reasoning taught in high school

In this level, students are ages from 12 to 18, usually divided into two di¤erent cycles (12-16 and 16-18, which is the case of the new Spanish education system, the one I know better). One of the most representative characteristics from an educational point of view is that students are in the adolescent phase: at that moment important physical and psychological changes can a¤ect their behaviour. To motivate many of these teenager students is sometimes a di¢cult

thing because they are in the phase of opposition to the "established world": parents, teachers, learning. I do not want to start a psychological debate, but I think that this is a very distinctive feature when teaching at that level.

The high school curriculum is usually structured in at least two di¤erent stages, the …rst one deals with common and basic areas such as Language, History, Mathematics, Sciences, etc., and the higher ones deal with more variate and speci…c subjects, such as Philosophy, Economics, Physics, Psychology, etc. In the …rst stage there is not any speci…c reference to Logic in the curriculum,. but it is an accepted thing that students somehow learn to reasoning logically in di¤erent areas of the curriculum. Mathematics seems to be the principal area where students have to learn logical reasoning. In the higher stages, specially when students have a course in Philosophy containing Logic (mainly propositional classical calculus at a very introductory level) they think they learn reasoning with Philosophy as well as with Mathematical courses. But from an instrumental point of view there is no doubt that Mathematics is the current context for logical reasoning in high school. In fact this is connected with learning /discovering inductive and deductive abilities.

2 Discovering inductive and deductive abilities in mathematical courses.

When I ask my students if learning Mathematics can be an enjoyable activity or if it is always a hard, painful thing, most of them answer : "yes, it can be enjoyable, but only if they (Mathematics) are understandable" and many of them recognize that the teacher is very important to understand Mathematics. What do students mean with understandable and thus enjoyable Mathematics?. When they are learning an algorithm in calculus or algebra or using a new mathematical language, they use expressions such as "it is easy", it is hard" or "when do I use it?", but in these cases they don't have to understand or not something. To understand Mathematics means to follow the underlying logical reasoning that allows them to construct a mathematical abstract concept or a solving strategy. Thus, it seems that to enjoy learning Mathematics is connected with using logical (deductive and inductive) abilities successfully.

Now, take these adolescent critical and insecure students and try them to learn how to develop inductive and deductive abilities in the context of Mathematics. It is often a very di¢cult thing. We must take into account that there is an intrinsic di¢culty in mathematical language and methods that are obstacles when exercising the "essay-error" method that develops the logical abilities. The abstract language of Mathematics, low motivation and the insu¢cient presence of the maths teacher in the process of learning reasoning (in Spain there are about 30 students per group) make understanding maths a di¢cult thing.

Learning some parts of Mathematics is not possible without using logical reasoning, but what I also defend here is that learning logical reasoning without mathematical language is possible and necessary. I think that the fact of teaching logical reasoning basically in mathematical courses is a special problem for the less brilliant students. When students have a previous structured logical way of thinking they are able to better learn Mathematics and it improves their capacity to better reasoning generally. But when students have to manage both with Mathematics and Logical Deduction, the difficulties increase and make Mathematics understandable and then only a hard thing to learn.

Now I shall make a sketch of the main parts of the mathematical curriculum connected with the logical/cognitive domain in secondary school.

- Processes of codification and decodification using numbers, graphics, functions or specific languages.
- Known conceptually natural numbers, integers, fractions, decimal and irrational numbers, its representation and operativity.
- Elaborate geometrical models by using plane and solid geometry.
- Study functions as a tool to describe information and to construct models.
- To use different mathematical languages in the appropriate situations.
- To use the mathematical way of reasoning.
- Discover the beauty of the mathematical structures.

When teaching these parts of Mathematics in high school there are many parts of Logic that are involved:

- The development of language and relations: verbal, symbolic, graphic. Syntaxis.
- Formalization of information in different languages.
- Using precision and rigor.
- Using connectives and inference rules (Modus Ponens, No contradiction) .
- Using restrictions and conditions (necessaries, sufficient)
- Representation of relations with diagrams.
- Construction of well defined concepts (primitive and derived concepts)
- Proofs and counterexamples.

- Using language and metalanguage.
- Paradoxes (as mathematical games).

All these logical concepts are also logical-mathematical concepts. At that level the border is very thin. But they are Logic concepts in their own right. From my point of view they are independent from the Mathematics concepts, they can be taught independently of Mathematics; and teaching Logic in high school would even improve the results when teaching Mathematics.

3 Inclusion of Logic in the curriculum of high school

In this section I present some ideas about how Logic might be brought into high school level. Before that, I would like to mention the importance of promoting and facilitating logical reasoning at an early age. If students are exposed to scholar experiences that involve abilities such as exploring, conjecturing and reasoning logically, as well as the ability to use a variety of mathematical methods to solve problems they will gain in "logical/mathematical power".

In high school level, an appropriate logical curriculum should consist in teaching the explicit logical notions and techniques to construct proofs and counterexamples, especially in the higher grades. In the …rst stage (12-16) the curriculum in Logic should contain :

- Formalization in di¤erent languages (also in algebraic languages). Work with connectives, negation, disjunction.
- Work on particularization and generalization.
- Work on relations with di¤erent representations. De…ning relations and sets.
- Study elementary concepts on Set Theory.
- Elementary Computation concepts: programs, languages, algorithms, etc.
- Give strategies to solve nonroutine problems (some of them with a mathematical context and some of them about everyday situations). Express with algorithms the strategy and whit precision the solution.
- Give proofs in very simple cases.

In higher stages (16-18) the stress must be on proof and deduction, and a possible curriculum could contain the following points:

- More Set Theory. Cardinality.
- Presentation of formal languages. (Elements of a formal language)
- Proofs. Elements of a logical proof. Examples with di¤erent languages. Also mathematical proofs as an important case.
- Construction of counterexamples.
- Introduction to Theoretical Computation.
- Metalanguage and paradoxes.

The more general goal of Logic in high school should be to understand the very nature of the subject as an ultimately deductive discipline and in the higher grades to understand the role of Logic in Mathematics and Computation.

In a report of the Committee on Logic and Education of the A.S.L two strategies were presented to work on a general recognition of Logic as a subject matter in its own right in the pre-college curriculum:

(1) "Work for a substantive addition to the Mathematics curriculum".

(2) "Work toward the day when all teachers will have an appreciation of Logic as an important discipline".

Both things are connected with a more general task: dispel the negative view of many of the academic and administrative colleagues that modern Logic is symbol pushing, and explain the necessity of Logic studies into the preparation of mathematical and computer high school teachers.