First International Congress on Tools for Teaching Logic Teaching Logic in Philosophy: Logical Language and Semantic Trees

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Abstract

Logic has ever been a philosophical discipline and a source of philosophical problems through the history of ideas, so that a base in logic is essential for education in any area of humanities. In this paper we point out why many faculties pay little attention to logic and propose two ways of teaching to regain that.

1 Logic and history of philosophy

Logic can be considered as a discipline that prepares theories of sound argumentation. In this sense logic was born with the great systems of philosophy but, has logic ever solved the methodological problems of sciences? Let us see: (a) Greek science, specially the mathematics, had developed forms of arguments that have been paradigmatic through the history, but Greek logic theories did not give account of that in an acceptable way. (b) During the Middle Age rhetoric becomes to be more relevant than logic. In Modern Age very little progress in developing a new logic to apply to important arguments took place. (c) From Boole, Frege, Russell, etc., logic adopted mathematical methods, then many philosophers did not pay attention to main issues that the new form of logic was stating, in front of them the logical empiricism was born under the banner of that new logic.

We can conclude that the absence of interest for teaching logic in philosophy may be an inheritance from an attitude that takes root in the history. In order to change that negative attitude, we must take the same references to show new perspectives: (a') The problem of existence of unsystematically conclusive arguments in Euclid's Elements, among others, has not been solved by logicians until they have adopted the mathematical method. (b*) Leibniz is an antecedent of logicism who thought to establish a lingua characteristica. Bolzano studied kind of propositions that was able to be represented as schemata and researched how the relation between premises and conclusion must be. (c*) Some works due to the parents of mathematical logic can be considered essential in contemporary philosophy, so Frege and Russell, for example, made to advance the philosophy of language and put the basis to develop analytical philosophy. We can also point out the importance of logicism, intuitionism and formalism.

Summary, logic was born as a child of philosophy to apply to mathematics and other sciences without success a lot of times, but when someone works in logic, whatever his or her point of view may be, he or she philosophizes somehow at last.

2 Grammatical structure and logical form

To distinguish logical form and grammatical form is not trivial: the man is mortal and Plato is mortal have the same grammatical form but its (associated) logical forms are very different, since in the first sentence it is stated that the concept "man" (the correspondent extension) is enclosed in (the extension of) the concept "mortal", while in the second the property "to be mortal" is attributed to Plato. So we have new arguments on account of learning logic in philosophical studies: logical consequence, a central notion in logic, concerns to meanings, and logical forms may not be intelligible through grammatical forms, therefore: a logical language is necessary, and the basic requirement should be that in its sentences the logical forms can be known without ambiguity. To obtain L we start from natural language (NL): if $\alpha \in NL$, (i) when α is atomic, $Par(\alpha) = \alpha$ (paraphrase of α), (ii) in other case, $Par(\alpha)$ is defined by means of paraphrases, introducing variables if necessary. Applying that to mentioned examples it is obtained for all x (if x is man, then x is mortal) and Plato is mortal. Then, considering subject and predicate as argument and function, both paraphrases can be rewritten respectively as follows: (x)(man(x) $\Rightarrow$ mortal(x)) and mortal(plato). Finally $L = \{Re(Par(\alpha)) : \alpha \in NL\}$, a hybrid between the natural language and a formal language.

After a number of exercises, to describe the semantics of L by means of extensional notions could be a new step. Then we can introduce an exact notion of first order formal language and define its semantics in terms of model theory. Now logical forms will be expressed as formulae: $\forall x(Mx \rightarrow Dx)$ and Dp, respectively. From that language an overview of the main parts of classical logic should be our objective. The symbol $\models$ will be as usual, emphasizing that (when "logical consequence") it has the important property of monotonicity: given the sets of formulae Γ and Δ, and the formula β, $\Gamma \models \beta \Rightarrow \Gamma \cup \Delta \models \beta$.

An advantage of this way of teaching is that to study some related areas will be easier for any student of philosophy: if you become familiar with that application of formal languages, you will have tools for representing knowledge and obtaining a theory of representation of discourse, for example. If L^* is a formal language so explained, we can define a "logic" as the following set: $Log(L^*) = \{\varphi \in L^* : \models \varphi\}$.

3 Semantic trees

The problem now is to find the formulae of L^* that form such logic (we should avoid to discuss the "decision problem" at the moment). Our proposal is tableaus or semantic trees method, due to Beth among others, a refutation procedure that is motivated by semantic concerns. Given correspondent rules (omitted to abbreviate), results to apply semantic trees can be proved:

Theorem 1 a formula $\alpha \in L^*$ is satisfiable if and only if its semantic tree is open.

$\Rightarrow$) by induction over the number of application of correspondent rules, ($\Leftarrow$) by defining a model with individual constants as universe of discourse and induction over rules (it satisfies all formulae of the open path) $\square$:

Theorem 2 Given $\alpha \in L^*$, $\neg\alpha$ is not satisfiable if and only if its semantic tree is closed

It is obtained from the former $\square$:

Corollary 3 Given $\alpha \in L^*$, $\models \alpha$ if and only if the semantic tree of $\neg\alpha$ is closed.

Since $\alpha_1, \ldots, \alpha_n \models \alpha \equiv \models \alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n \rightarrow \alpha$, semantic trees can also be applied to verify whether logical consequence exists. The method can be extended to study formulae of de kind $\forall x \exists y \beta$ (no bound variables in β) that generate infinite trees but are satisfiable in finite domains, by modifying the rule for $\exists$: from $\exists x \beta$, if $a_1, a_2, \ldots, a_n$ are the individual constants that occur in correspondent path, add $\beta(a_1/x) \vee \beta(a_2/x) \vee \ldots \vee \beta(a_{n+1}/x)$. This treatment, proposed by Boolos and Díaz independently, permit us sometimes to find a minimal class of models. We can also apply that to second order logic, for example to study Σ^1_1-formulae and Π^1_1-formulae.

Natural deduction (represented by $\vdash$) is often displayed as a syntactic approach to inference. Then $Cal(L^*) = \{\beta \in L^* : \vdash \beta\}$. Besides the usual system, another system of natural deduction ($\vdash_{st}$) can be defined, based in semantic trees; every rule for semantic trees is taken as a rule, the following refutation rule is important: $\beta \vdash_{st} \bot \Rightarrow \vdash_{st} \neg\beta$.

Theorem 4 Given $\beta \in L^*$, the semantic tree of β is closed if and only if $\vdash_{st} \neg\beta$.

It is derived from definitions $\square$:

Corollary 5 Given $\beta \in L^*$, $\models \beta$ if and only if $\vdash_{st} \neg\beta$.

Theorem 6 For every ¯ 2 L¤, ` ¯ if and only if `$_{st}$ ¯.

In short, some rules belong to both systems, and it can easily be proved that every speci…c rule of a system is derived in the other one. ¤:

Corollary 7 ¯ 2 Cal(L¤) if and only if the semantic tree of :¯ is closed.

Corollary 8 Cal(L¤) = Log(L¤).

Summarizing, the use of semantic trees in that contexts is good for several reasons:

- it provides an excellent base from which to teach logic combining clarity and rigour,
- from that you will make easy to learn formal methods and its application in philosophical works (philosophical analysis, metalogical problems, etc.),
- because of its constructive nature, it can appear in a computational setting, so it can help to consider computer sciences, and related disciplines, from a philosophical point of view, as well as to implement and develop programs which may be used for teaching logic.